A Threshold Enhancement Technique for Chaotic On-Off Keying Scheme

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Abstract

In this paper, an improvement for Chaotic ON-OFF (COOK) Keying scheme is proposed. The scheme enhances Bit Error Rate (BER) performance of standard COOK by keeping the signal elements at fixed distance from the threshold irrespective of noise power. Each transmitted chaotic segment is added to its flipped version before transmission. This reduces the effect of noise contribution at correlator of the receiver. The proposed system is tested in Additive White Gaussian Noise (AWGN) channel and compared with the standard COOK under different E_b/N_o levels. A theoretical estimate of BER is derived and compared with the simulation results. Effect of spreading factor increment in the proposed system is studied. Results show that the proposed scheme has a considerable advantage over the standard COOK at similar average bit energy and with higher values of spreading factors.

Keywords: Chaotic Communication, Error Probability, Correlation, Sequences, Gaussian Distribution.

1. INTRODUCTION

Conventional digital communication techniques mainly rely on deterministic carrier signal such as sinusoidal in the modulators. Recently, chaotic signals are considered as a potential alternate for the sinusoidal carriers mainly owing to their low cross correlation value and impulse like autocorrelation properties. For this purpose, many communication schemes have been developed to utilize the properties of the chaotic signals. One such example is Chaotic Shift Keying (CSK) systems [1].

CSK systems are divided into two categories, namely coherent CSK and non-coherent CSK systems [2]. The coherent CSK was first proposed by Partiz [3]. The idea is to encode each different information bit with a chaos basis function, where the receiver should generate the same chaotic signal of the transmitter for demodulation. Several techniques have been investigated to improve the BER performance of coherent CSK. By making the chaotic source more Gaussian to match the Probability Density Function (PDF) of the noisy channel, BER of coherent CSK is improved [4]. In [5], Space-Time-Block-Code (STBC) is suggested to enhance the performance of coherent CSK. Each chaotic segment has low cross correlation value with its shifted version, and when each version is assigned to a single user, bandwidth efficient multiuser scheme is established [1]. Performance evaluation and theoretical expression for multiple user CSK system are introduced in [6]. Another multiple user's scheme which uses maximum likelihood detection to

minimize BER is described in [7]. In all previous literature of coherent CSK, it is assumed that the transmitter and receiver generators are synchronized, and the receiver is able to generate a copy of the transmitter sequence. This is a difficult task, theoretically not possible at noisy channels due to the chaotic nature of the source [1].

In order to overcome the synchronization problem, a non-coherent version of CSK is initially suggested by [8]. Detection is done by an estimation of the received signal energy [8] or by finding the generated map using return map regression technique as in [9]. In non-coherent energy based CSK, chaotic signals with different energies profile are used to represent the binary symbols. If binary "1" is to be sent, a chaotic signal with average bit energy E_1 is transmitted. If "0" is to be sent, signal with average bit energy E_0 is transmitted. A simple and alternate approach of non- coherent energy based CSK named as Chaotic ON-OFF keying system (COOK) is described and compared with other modulation techniques in [8], where the chaotic signals are emitted from the source in the bit duration of symbol '1', otherwise, no transmission is taken place within the bit duration of symbol '0'.

At the receiver of non-coherent energy based CSK systems, a simple energy estimator is used for information detection. By squaring and integrating process, each received signal, which includes a noise term contribution, is multiplied by itself and averaged over one bit duration. A comparator with a predetermined threshold is set to decode the information bits. However, threshold is depending on the Signal-to-Noise Ratio (SNR) which leads to the Threshold Shift Problem (TSP) [2].

In this paper, a time-reversal technique, which utilizes the low correlation value between the chaotic segment and its flipped version, is used to overcome TSP and is used to enhance BER performance of COOK by maintaining a threshold in the mid-way between the signal elements and is independent of the noise value. Time reversal scheme [10] finds a wide application in the field of wave propagation and imaging in random media. It is proved in [11, 12] that such time reversal scheme results in self-averaging and statistical stability. This technique can be applied effectively in chaotic communication scenario as well, primarily due to the statistical behavior or random nature of chaotic signals. Section 2 will briefly explain the structure of COOK. The proposed system structure is discussed in 3. Theoretical estimation for the BER is derived in section 4. Simulation results for different scenarios are discussed in section 5.

2. CHAOTIC ON-OFF KEYING SCHEME; THRESHOLD OPTIMIZATION PROBLEM

Transmitter structure of COOK is illustrated in Fig. 1. Binary '1' is presented by continuous transmission of *M* chaotic signal samples x_i (i.e. chaotic segment) within single bit duration. Bit '0' will be presented by null transmission within the same duration. Thus, the transmitted signal s_i can be written as:

$$s_i = \alpha \ b_l \ x_i \qquad M \ (l-1) < i \le l \ M \tag{1}$$

where bit $b_1 \in \{1, 0\}$ and α is the gain factor.



FIGURE 1: COOK Transmitter.

Signal energy values for binary transmission is given by

$$E_0 = 0;$$

$$E_1 = \alpha^2 E(\sum_{i=1}^M x_i^2) at \ \alpha = \sqrt{2} \xrightarrow{\text{yields}} E_1 = 2M \ \sigma_x^2;$$

Where E(.) is the expected value operator and σ_x^2 is the average signal power. Average bit energy is given by

$$E_b = (E_0 + E_1) / 2 = M \sigma_x^2$$

Receiver structure of conventional COOK is illustrated in Fig. 2. Each incoming segment is multiplied with itself and averaged over one bit duration. Integrator output is compared with predetermined threshold λ_{th} .

Let us make the standard assumption that the received signal r_i sample is given by $r_i = s_i + \psi_i$ where ψ_i is a Gaussian noise sample with $E(\psi_i) = 0$. Thus, the output of the correlator Z at the end of bit duration l can be written as

$$Z_{l} = \sum_{i=(l-1)M+1}^{lM} r_{i} r_{i}$$

$$Z_{l} = b_{l} \alpha^{2} \sum_{i=(l-1)M+1}^{lM} x_{i}^{2} + 2b_{l} \alpha_{l} \sum_{i=(l-1)M+1}^{lM} x_{i} \psi_{i} + \sum_{i=(l-1)M+1}^{lM} \psi_{i}^{2}$$

For single bit (i.e. l = 1), Z_l can be simplified to

$$Z = b \alpha^{2} \sum_{i=1}^{M} x_{i}^{2} + 2b \alpha \sum_{i=1}^{M} x_{i} \psi_{i} + \sum_{i=1}^{M} \psi_{i}^{2}$$
(2)

First part of the (2) contains signal energy. Second part is the signal-noise interference component, since x and are ψ statically independent, it can be verified that the second part is approximately zero for large value of M, while the last part represents the noise power.



Assume that the receiver has no information about the noise level, then threshold λ_{th} can be set only at the mid distance between the signal elements and it is given by [1]

$$\lambda_{th} = (E_0 + E_1) / 2 = \frac{1}{2} \alpha^2 M \sigma_x^2 = E_b$$

Bit decoding is conducted by comparing Z with λ_{ih} . At moderate and low SNR, the correlator output may cross the theoretical value of the threshold even no signal was transmitted. Fig. 3 shows the histogram of the correlator output at $E_b / N_o = 18 \text{ dB}$ and with M = 10. It can be noticed clearly that the signal elements are shifted by the amount of noise power with respect to the theoretical value of the threshold.



FIGURE 3: Histogram of noisy received signal in non-coherent COOK receiver with M=150 and at $E_{h} / N_{a} = 18 \text{ dB}$.

3. PROPOSED SCHEME: FLIPPED-CHAOTIC ON-FF KEYING (FCOOK) 3.1 Transmitter Description

The transmitter setup for the proposed scheme is shown in Fig. 4. Chaos generator generates the chaotic segment $\{x_i^1\}$ with the length of *M*. To generate the time reversal sequence, each sample of $\{x_i^1\}$ is stored and timely reversed over one bit duration to generate the second set of sequence $\{x_i^2\}$ such that $x_i^2 = x_{Ml-imod(M)+1}^1$. For example, if the chaotic segment $\{x_{11}^1, x_{12}^1, x_{13}^1, \dots, x_{20}^1\}$ is used to modulate the second information bit (l = 2) at spread factor M=10, then the corresponding time reversal sequence is given by $\{x_{11}^2, x_{12}^2, x_{13}^2, \dots, x_{20}^2\} = \{x_{20}^1, x_{19}^1, x_{18}^1, \dots, x_{11}^1\}$. Thus, at the end of the M clock cycle, $\{x_i^2\}$ of block size M is available which is the time-reversed sequence of $\{x_i^1\}$.

An initial delay of M samples is included to compensate the delay incurred in time reversal of M values generation. This delay is ignored in our calculation because they are equal in both arms. The control circuit switches to ON and OFF according to the transmitted bit. If binary '1' is sent, the transmitted signal will be the sum of the chaotic segment and its time flipped version. Otherwise, null transmission is taken place in the same bit duration. Then, the transmitted signal at the ith instant can be written as

$$s_{i} = b_{l} \left(x_{i}^{1} + x_{i}^{2} \right) = b_{l} \left(x_{i}^{1} + x_{Ml-imod(M)+1}^{1} \right)$$

Where $(l-1)M \leq i \leq lM$ and $b_i \in \{1,0\}$



FIGURE 4: FCOOK Transmitter Structure.

3.2 Receiver Description

The receiver block diagram is presented in Fig. 5. Each received segment $\{r_i\}$ is stored and flipped to get its time reversed form $\{r_i\}$ in a similar way as that described in the transmitter section. This time reversed version is then multiplied with the received signal r_i . As in the transmitted part, the delay by M samples is included to compensate the delay incurred in storing and reversing of M values. The product is then integrated over one bit period and is passed through a threshold circuit to decide whether the transmitted bit is a ONE or ZERO.



FIGURE 5: Receiver Structure.

Assume the incoming signal is received via AWGN channel, and then the received signal r_i can be written as

$$r_i = s_i + \psi_i = b_l x_i^1 + b_l x_{Ml-imod(M)+1}^1 + \psi_i$$

and

$$r'_{i} = b_{l} (x_{Ml-imod(M)+1}^{1} + x_{i}^{1}) + \psi_{Ml-imod(M)+1}$$

The correlator output can be given as

$$Z_{l} = \sum_{i=(l-1)M+1}^{lM} r_{i} r_{i}'$$

For simplicity and without loss of generality, we can consider the correlator output for single bit duration and in terms of x_i , Z_i can be rewritten as

$$Z = \sum_{i=1}^{M} r_{i} r_{M-i+1} \qquad 0 < i \le M$$

$$Z = \sum_{i=1}^{M} (bx_{i} + bx_{M-i+1} + \psi_{i}) (bx_{M-i+1} + bx_{i} + \psi_{M-i+1})$$

$$Z = b \sum_{i=1}^{M} (x_{i}^{2} + x_{M-i+1}^{2}) + 2b \sum_{i=1}^{M} (x_{i} x_{M-i+1}) + 2b \sum_{i=1}^{M} (x_{i} \psi_{i}) + 2b \sum_{i=1}^{M} (x_{i} \psi_{M-i+1}) + 2b \sum_{i=1}^{M} (x_{i} \psi_{i}) + \sum_{i=1}^{M} (\psi_{i} \psi_{M-i+1})$$
(3)

FCOOK signal energy is represented by the first term which will have fluctuated value due to the chaotic nature of the source. The correlation between the chaotic segment and its time flipped version over finite sequence length is formulated by the second term. Remaining terms are signal-noise or noise-noise terms with random quantity having zero mean and can contribute slightly to the correlator positively or negatively. For the last term, which represent correlation between the received noise segment and its corresponding time reversed version, noise contribution to the correlator output will be largely reduced compared to that in (2) due to very low correlation value between each noise segment and it flipped version. This identifies the major contribution of this work. However, the scheme will have more intra-signal terms with respect to COOK. The problem can be solved by using larger values of spreading factor *M*.

The output of the correlator is applied to a decoding circuit. Decoder is based on the following rule

$$\hat{b} = \begin{cases} 0 & Z < \lambda_{th} \\ 1 & Z \ge \lambda_{th} \end{cases}$$

4. PERFORMANCE EVALUATION

can be calculated by

Baseband Model and Gaussian approximation (GA) method are used to derive BER expression. The method is valid for large spreading factor [13, 14]. To proceed with the evaluation, the following assumptions are considered

- 1. Chaotic signal x_i is stationary (which is a standard in chaotic systems). Hence, x_{M-i+1} is also stationary. It can be easily verified that $E(x_i \cdot x_{M-i+1}) \approx 0$ for large value of M [2]. 2. Chaotic signal x_i is statistically independent from Ψ_j for any (i, j). Furthermore, Ψ_i is
- statistically independent from Ψ_i for any $i \neq j$.
- 3. A Symmetric tent map, which is given by the equation, $x_{n+1} = 1 2|x_n|$, is used to generate chaotic sequence x_i where x_i is uniformly distributed from [-1,1] with an average value of zero [15].

Based on the assumptions 1 and 2, Z tends to have Gaussian distribution particularly at large value of M [16]. Therefore, it is sufficient to calculate the mean and average of Z to evaluate the performance.

Let (μ_0, σ_{z_0}) and (μ_1, σ_{z_1}) are the average values and variances of the observation variable when binary '1' and '0' are sent. Therefore, the two conditional density function of the correlator output Z given as input to the decision device is given by:

$$P(Z/1) = \frac{1}{\sqrt{2\pi} \sigma_{Z1}} e^{\frac{-(Z-\mu_1)^2}{2\sigma_{Z1}^2}}$$
$$P(Z/0) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z-\mu_0)^2}{2\sigma_{Z0}^2}}$$

$$\sqrt{2\pi} \sigma_{z_0}$$

From (3), the expected values and variance of the observation variable Z when binary '1' is send

$$\mu_{1} = E(Z/1) = E[\sum_{i=1}^{M} (x_{i}^{2} + x_{M-i+1}^{2})]$$

$$= \sum_{i=1}^{M} E(x_{i}^{2} + x_{M-i+1}^{2})$$

$$= 2\sum_{i=1}^{M} E(x_{i}^{2}) = 2M\sigma_{x}^{2} = 2E_{b}$$

$$\delta_{Z1} = Var(Z/1)$$

$$= Var\left[\sum_{i=1}^{M} (x_{i}^{2} + x_{M-i+1}^{2})\right] + Var\left[2\sum_{i=1}^{M} (x_{i} \cdot x_{M-i+1})\right] + Var\left[2\sum_{i=1}^{M} (x_{i} \cdot \psi_{i})\right] + Var\left[2\sum_{i=1}^{M} (x_{i} \cdot \psi_{M-i+1})\right] + Var\left[\sum_{i=1}^{M} \psi_{i} \cdot \psi_{M-i+1}\right]$$

$$= A + B + C + D + E$$

Due to symmetry in the term, A can be rewritten as

$$\begin{split} A = Var \left[\sum_{i=1}^{M} (x_i^2 + x_{M-i+1}^2) \right] \\ = Var \left[2 \sum_{i=1}^{M/2} (x_i^2 + x_{M-i+1}^2) \right] \\ = 4 \frac{M}{2} Var \left[(x_i^2 + x_{M-i+1}^2) \right] \\ = 2M \left[Var \left(x_i^2 \right) + Var \left(x_{M-i+1}^2 \right) + 2 \cos(x_i^2, x_{M-i+1}^2) \right] \\ Var \left(x_i^2 \right) = E \left(x_i^4 \right) - (E \left(x_i^2 \right))^2 \\ E \left(x_i^4 \right) = \frac{1}{2} \int_{-1}^{+1} x^4 dx = \frac{1}{2} \left[\frac{x^3}{5} \right]_{-1}^{-1} = \frac{1}{5} \\ E \left(x_i^2 \right) = \frac{1}{2} \int_{-1}^{+1} x^2 dx = \frac{1}{2} \left[\frac{x^3}{3} \right]_{-1}^{-1} = \frac{1}{3} \\ A = 2M \left[\frac{1}{5} - \left(\frac{1}{3} \right)^2 \right] = \frac{2M.4}{45} = \frac{8M}{5.9} = \frac{8}{5} M \sigma_x^2 \sigma_x^2 \\ A = \frac{8}{5} \frac{Eb^2}{M} \\ B = 2 \sum_{i=1}^{M} (x_i \cdot x_{M-i+1}) = 4 \sum_{i=1}^{M/2} (x_i \cdot x_{M-i+1}) \\ Var (B) = 16 \sum_{i=1}^{M/2} Var |E(x_i)|^2 Var(x_{M-i+1}) + 16 \sum_{i=1}^{M/2} |E(x_{M-i+1})|^2 Var(x_i) + 16 \sum_{i=1}^{M/2} Var(x_i) Var(x_{M-i+1}) \\ = 0 + 0 + 16 \sum_{i=1}^{M/2} Var(x_i) Var(x_{M-i+1}) = 8M \sigma_x^2 \sigma_x^2 = 8 \frac{Eb^2}{M} \\ C = Var \left[2 \sum_{i=1}^{M} (x_i \cdot \psi_i) \right] \\ = 4 \left[\sum_{i=1}^{M} Var(x_i \cdot \psi_i) \right] = 4M \sigma_x^2 \sigma_0^2 \end{split}$$

Let $N_{_0}=2\sigma_{_0}^2$ where $N_{_0}$ the power spectral density. Therefore,

$$C = 2E_b N_o$$

Similarly

$$D = 4M\sigma_x^2\sigma_0^2 = 2E_bN_0$$

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For noise-noise term

$$E = 4\frac{M}{2}\sigma_0^2\sigma_0^2 = 2M\frac{N_0}{2}\frac{N_0}{2} = \frac{MN_0^2}{2}$$
$$\therefore \sigma_{Z1}^2 = A + B + C + D + E$$
$$\sigma_{Z1}^2 = \frac{8}{5}\frac{E_b^2}{M} + 8\frac{E_b^2}{M} + 4M\sigma_x^2\sigma_0^2 + 4M\sigma_x^2\sigma_0^2 + \frac{MN_0^2}{2}$$
$$\sigma_{Z1}^2 = \frac{48}{5M}E_b^2 + 4E_bN_0 + \frac{MN_0^2}{2}$$

Similarly we can show that

$$\mu_0 = 0$$
 and $\sigma_{Z0}^2 = \frac{M N_0^2}{2}$

A typical plot of the histogram for the FCOOK correlator output is given in Fig. 6. Clearly, it can be noticed that the threshold lies in the midpoint between signal elements irrespective of noise contribution. In other words, average value of the signal element has fixed position with respect to the theoretical threshold.

Since a received bit '1' can be detected as bit '0' by the receiver, if the input to the decision device is less than the threshold. While bit '0' can be detected as bit'1' if the metric is more than the threshold and assumes that $P_r(0) = P_r(1)$.



FIGURE 6: FCOOK Histogram of noisy received signal in non-coherent FCOOK receiver with M=150 and E_b / N $_0$ =18 dB.

Then, the average P_e can be expressed as

$$P_{e} = \frac{1}{2} \left(\frac{1}{\sqrt{2\pi} \sigma_{Z1}} \int_{-\infty}^{\lambda_{th}} e^{\frac{-(Z-2Eb)^{2}}{2\sigma_{Z1}^{2}}} dZ + \frac{1}{\sqrt{2\pi} \sigma_{Zo}} \int_{\lambda_{th}}^{\infty} e^{-\frac{Z^{2}}{2\sigma_{Z0}^{2}}} dZ \right)$$

$$P_e = \frac{1}{2} \left(Q \left(\frac{E_b}{\sigma_{Z_1}} \right) + Q \left(\frac{E_b}{\sigma_{Z_0}} \right) \right)$$

where Q is the Q function defined by

$$Q(a) = \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-\frac{y^2}{2}} dy$$

$$P_{e} = \frac{1}{2} \left(Q \left(\sqrt{\frac{E_{b}^{2}}{5M}^{2} + 4EbN_{0} + \frac{MN_{0}^{2}}{2}}} \right) + Q \left(\sqrt{\frac{E_{b}^{2}}{MN_{0}^{2}}} \right) \right)$$

$$P_{e} = \frac{1}{2} \left(Q \left(\sqrt{\left(\frac{48E_{b}^{2}}{5M}^{2} + 4E_{b}N_{0} + \frac{MN_{0}^{2}}{2}\right)^{-1}} \right) + Q \left(\frac{E_{b}}{N_{0}} \sqrt{\frac{2}{M}}\right) \right)$$

$$P_{e} = \frac{1}{2} \left(Q \left(\sqrt{\frac{E_{b}}{N_{0}} \left(\frac{48E_{b}}{5MN_{0}} + 4 + \frac{MN_{0}}{2Eb}\right)^{-1}} \right) + Q \left(\frac{Eb}{N_{0}} \sqrt{\frac{2}{M}}\right) \right)$$
(4)

5. RESULTS AND DISCUSSION

The performance of the proposed system in AWGN channel is compared with the standard COOK at various values of spreading factors and under different levels of E_b / N_0 . To have equal average bit energy for both systems, each generated signal in COOK is multiplied by $\sqrt{2}$ (i.e. $\alpha = \sqrt{2}$). Therefore, Average bit energy, under the assumption that P (0) =P (1), can be given as $E_{b-cook} = (0 + \alpha^2 M \sigma_x^2)/2 = M \sigma_x^2$ which is equal to $E_{b-FCOOK}$. Correlator Threshold λ_{th} is set at the mid-way between the signal elements for both systems such that $= \alpha^2 M \sigma_x^2/2 = M \sigma_x^2$. In other words,. The theoretical expression for BER is derived and compared with the simulation results. In addition, the effect of spreading factor increment on the system performance at fixed E_b / N_0 is studied.

At lower values of M, COOK system has very poor and not practical performance as shown in [17]. A comparison between FCOOK and COOK systems at M=100,120 and 150 is shown in Fig 7.a. It can be noticed that the FCOOK outperforms COOK at moderate E_b / N_0 levels due to the reduction of noise effect at the output of FCOOK correlator. At higher E_b / N_0 , FCOOK has less performance than COOK. This is due to the negligible effect of the noise-noise terms in (2), Hence the threshold will be almost in the middle between signal elements. Additionally, non-complete orthogonal in signal-signal and signal-noise terms in (3) for the proposed system can slightly decrease the performance compared to COOK. When the spreading factor is increased to M=200, 300 and 500 as shown in Fig. 7.b, FCOOK can achieve BER of 1 x 10⁻⁴ at 18 dB while COOK system cannot operate due to signal elements shift.

Effect of spreading factor is illustrated in Fig. 8. Simulations results of the BER is compared with the estimated value in (4) at $E_b / N_0 = 10$ dB and 17dB respectively. When spreading factor ranges from 5 to 50, BER is low due to variable average bit energy and non-complete orthogonality of the signal-signal and signal-noise terms. When M is increased, keeping E_b / N_0 constant, performance is enhanced due to the orthogonality enhancement of intra-signal terms, typically at M = 100, then the performance starts to degrade. The reason for this degradation is that if we increase the spreading factor M keeping E_b / N_0 constant at fixed value, there is an increase in N_o proportional to M, Therefore, while useful signal linearly increases with M and so does the standard deviation of signal –signal and signal-noise terms in (3), the standard deviation of $\sum (\Psi_i \cdot \Psi_{M-i+1})$ grows faster with higher the value of M and result in increased BER.



FIGURE 7.a: BER performance comparison between COOK and FCOOK at M =100, 120 and 150.



FIGURE 7.b: BER performance comparison between COOK and FCOOK at M =200, 300 and 500.



FIGURE 8: Simulation results and analytical estimate for BER performance of FCOOK under different spreading factors and fixed E_b / N_0 .



FIGURE 9: Simulation results and analytical estimate for BER performance of FCOOK under different E_{h} / N_{0} and at M=150,300 and 500.

6. CONCLUSION

In this paper, a new non-coherent energy based chaos communication scheme is proposed. The system is based on adding each emitted chaotic segment with this timely reversed version over one bit duration. At the receiver, each incoming signal is correlated with its flipped version also. This prevents noise power contribution to the correlator output and reduces the effect of threshold shift problem. The system is tested in AWGN channel and compared with standard COOK at different values of spreading factor. Simulation Result shows that the proposed scheme has a reasonable advantage over the standard COOK at higher spreading factors.

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